

Investigation of Effect on Stresses in Rotating Disc with Non Central Circular Holes

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ABSTRACT:

In the present work the problem of circular disc with a central hole and a symmetrical array of non-central holes subjected to rotation are analysed using Finite Element Method. The Stress Concentration Factors (SCF) is derived for various geometric parameters like, R_2/R_1 , $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N). It is seen that, as the number of holes increases, the SCF decreases. The results are compared with the analytical solution given by H.E.Ang and C.L.Tan.

KEYWORDS: Disc, Non central hole, Stress concentration, FEM

NOTATION

A, B, C - positions at edges of holes shown in fig.1.

R_1 - Radius of central hole (bore) of disc

R_2 - Outer radius of disc

R_b - Pitch circle radius of holes

d - Hole diameter

N - Number of holes

E - Modulus of Elasticity

ν - Poisson's ratio

ρ - Density of material

ω - Angular velocity of disc

σ_θ - Hoop stress or tangential stress

σ_r - Radial stress

$(\sigma_\theta)_A$ - Hoop stress at point A

$(\sigma_\theta)_B$ - Hoop stress at point B

1. INTRODUCTION

There are machines elements which rotate while performing the required functions. These include flywheels, thin rings, circular discs, pulley rims, cylinders and spherical shells. Due to rotation, centrifugal stresses are developed in these elements. Rotating disc shown in fig.1 found in numerous industrial applications. High centrifugal stresses, radial and hoop stresses occur at the non- central holes needed to bolt discs together rotating at high speed, such as compressor and turbine rotors of aircraft engines, flywheel, gears, etc.

In the present work the problem of circular disc with a central hole and a symmetrical array of non-central holes subjected to rotation is analysed by using Finite Element Method (FEM).

The finite element approach is used to evaluate the stresses in the rotating disc with central hole by varying R_2/R_1 , $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N) and to study the effect of variation of these parameters in the stress concentration factor in the disc.

2. INTRODUCTION TO PROBLEM

In this project, effect of non-central holes in disc with central hole is investigated considering various geometrical parameters of the disc. The disc is shown in fig.1.

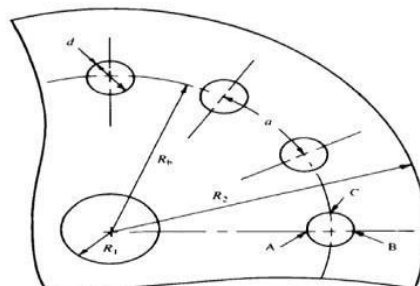


Fig.1. Circular disc with a central hole and a symmetrical array of non-central holes

The various geometric parameter ratios & their variation are as follows,
 $R_2 / R_1 = 3, 4, 5, 6, 7, 8, 9, 10$

$d / 2R_1 = 0.025, 0.05, 0.075, 0.1, 0.125, 0.15, 0.175, 0.2, 0.225, 0.25$

$R_b / (R_2 - R_1) = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$

$N = 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 28, 32, 36, 40$

The disc is made up of steel and its mechanical properties are tabulated in Table.1.

Modulus of elasticity (MPa)	E	200x103
Poisson's Ratio	v	0.3
Density of material (Kg/mm ³)	ρ	7800

Table 1: Properties of Material

3. VARIATION OF SCF WITH RESPECT TO GEOMETRICAL PARAMETERS EVALUATED BY FE APPROACH

Principal stress contours for some ratios R_2/R_1 , $R_b / (R_2-R_1)$, $d/2R_1$, N are shown in fig.2 to fig.5 as an illustration.

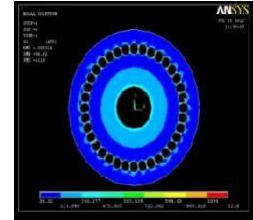
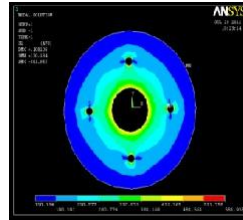
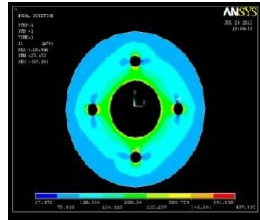
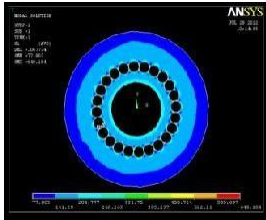


Fig.2: Principal Stress Contour (σ_θ) for $R_2/R_1=3$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.25$, $N=24$

Fig.3: Principal Stress Contour (σ_θ) for $R_2/R_1=3$, $R_b/(R_2-R_1)=0.9$, $d/2R_1=0.225$, $N=4$

Fig.4: Principal Stress Contour (σ_θ) for $R_2/R_1=4$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.2$, $N=4$

Fig.5: Principal Stress Contour (σ_θ) for $R_2/R_1=4$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.2$, $N=36$

Contour (σ_θ) for $R_2/R_1=3$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.25$, $N=24$

Contour (σ_θ) for $R_2/R_1=3$, $R_b/(R_2-R_1)=0.9$, $d/2R_1=0.225$, $N=4$

Contour (σ_θ) for $R_2/R_1=4$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.2$, $N=4$

Contour (σ_θ) for $R_2/R_1=4$, $R_b/(R_2-R_1)=0.8$, $d/2R_1=0.2$, $N=36$

An effort is made to show the variation of SCF with respect to geometrical parameter of disc i, e, $R_b/(R_2-R_1)$, R_2/R_1 , $d/2R_1$ and number of holes (N). These variations are shown in Fig.6 to Fig. 13.

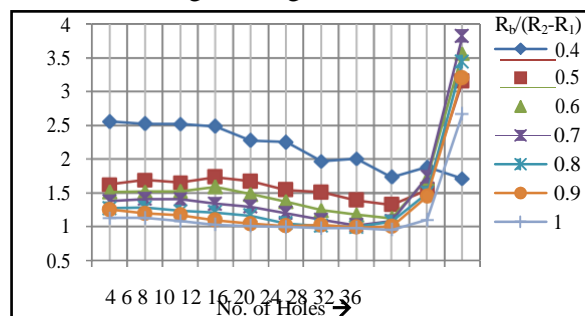
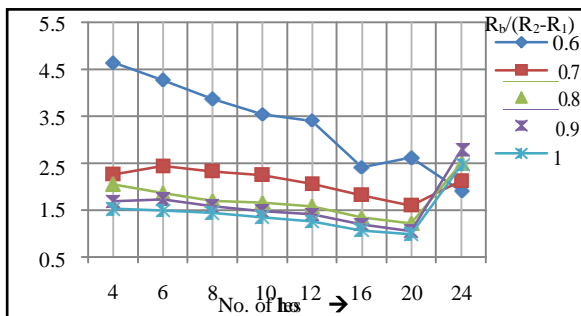


Fig.6: Variation of SCF with Respect to Number of Holes for $R_2/R_1=3$

Fig.7: Variation of SCF with Respect to Number of Holes for $R_2/R_1=4$

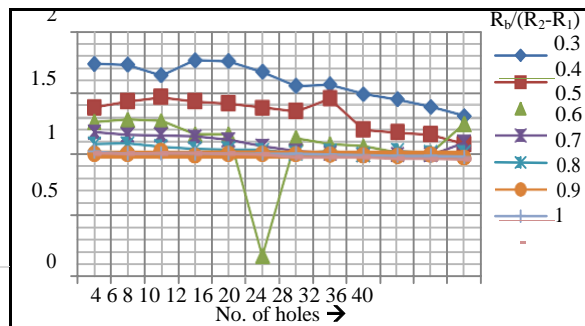
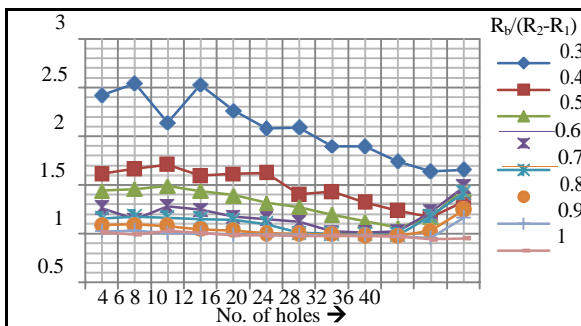


Fig.8: Variation of SCF with Respect to Number of Holes for $R_2/R_1=5$

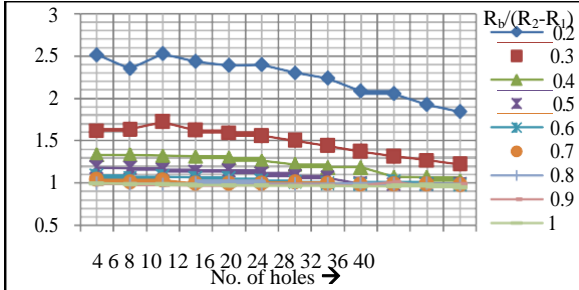


Fig.9: Variation of SCF with Respect to Number of Holes for $R_2/R_1=6$

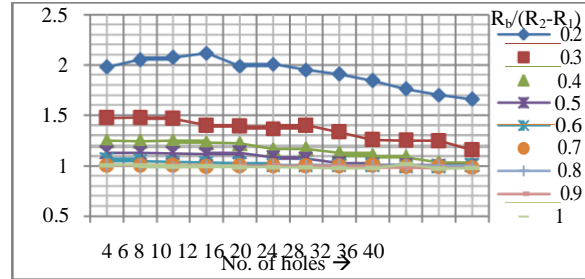


Fig.10: Variation of SCF with Respect to Number of Holes for $R_2/R_1=7$

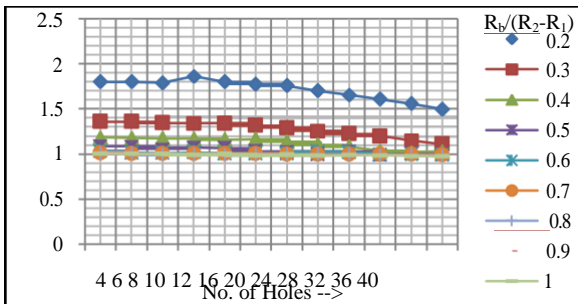


Fig.11: Variation of SCF with Respect to Number of Holes for $R_2/R_1=8$

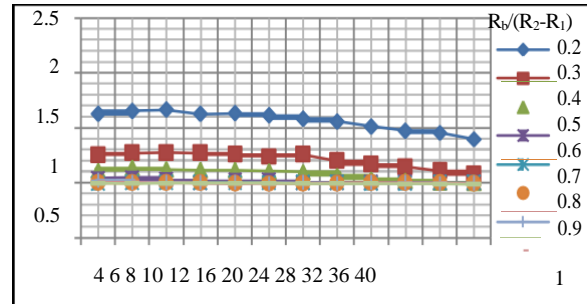


Fig.12: Variation of SCF with Respect to Number of Holes for $R_2/R_1=9$

4. COMPARISON OF FE RESULTS WITH ANALYTICAL RESULTS REPORTED BY H.E. ANG & C.L.TAN

The physical geometries and SCF given by H.E.ANG and C.L.TAN and evaluated by FE approach are given in Table 2 and Table 3 for the ratio of $d/2R_1=0.05$ and $d/2R_1=0.075$.

R_2/R_1	$R_b/(R_2-R_1)$	R_b	$d/2R_1$	d	N	$\sigma_{\theta} _{FEM}$ pt. A (MPa)	$\sigma_{\theta} _{THEO}$ (MPa)	K_A	$K_A _{The.}^s$
3	0.6	24	0.05	2	20	485.858	260.032	1.868	1.243
					24	498.441	260.032	1.916	1.220
					28	471.526	260.032	1.813	1.197
					32	476.938	260.032	1.834	1.174
					36	460.247	260.032	1.769	1.151
					40	460.83	260.032	1.772	1.128
	0.7	28	0.05	2	20	464.486	260.032	1.786	1.163
					24	436.965	260.032	1.680	1.143
					28	471.523	260.032	1.813	1.124
					32	414.451	260.032	1.593	1.104
					36	397.778	260.032	1.529	1.084
					40	390.733	260.032	1.502	1.065
	0.8	32	0.05	2	20	402.29	260.032	1.547	1.080
					24	397.121	260.032	1.527	1.063
					28	373.352	260.032	1.435	1.047
					32	365.164	260.032	1.404	1.031
					36	354.958	260.032	1.365	1.015
					40	343.583	260.032	1.321	0.998
0.5	30	0.05	2	20	721.946	457.622	1.577	1.192	
				24	699.263	457.622	1.528	1.176	
				28	706.564	457.622	1.543	1.160	

4	0.6	36	0.05	2	32	714.737	457.622	1.561	1.143
					20	685.121	457.622	1.497	1.099
					24	625.976	457.622	1.367	1.086
					28	656.061	457.622	1.433	1.074
					32	622.804	457.622	1.360	1.061
					36	602.582	457.622	1.316	1.049
	0.7	42	0.05	2	40	572.771	457.622	1.251	1.036
					20	608.248	457.622	1.329	1.017
					24	599.54	457.622	1.310	1.007
					28	560.41	457.622	1.224	0.997
					32	541.045	457.622	1.182	0.9876
					36	530.183	457.622	1.158	0.9778
	0.8	48	0.05	2	40	528.798	457.622	1.155	0.968
					20	552.879	457.622	1.208	0.9425
					24	547.907	457.622	1.197	0.9342
					28	546.596	457.622	1.194	0.9260
					32	537.205	457.622	1.173	0.9177
					36	509.025	457.622	1.112	0.9095
				40	490.968	457.622	1.072	0.9013	

Table 2: Comparison of FE and Analytical Results for $d/2R_1=0.05$

R_2/R_1	$R_b/(R_2-R_1)$	R_b	$d/2R_1$	d	N	$\sigma_0 _{FEM}$ pt. A (MPa)	$\sigma_0 _{THEO.}$ (MPa)	K_A	$K_A _{The.}$
3	0.6	24	0.075	3	20	485.858	260.032	1.8684	1.2477
					24	498.441	260.032	1.9168	1.2081
	0.7	28	0.075	3	20	464.486	260.032	1.7862	1.153
					24	436.965	260.032	1.6804	1.1174
	0.8	32	0.075	3	20	402.29	260.032	1.5470	1.0751
					24	397.121	260.032	1.5271	1.0448
4	0.5	30	0.075	3	20	721.946	457.622	1.5776	1.1658
					24	699.263	457.622	1.5280	1.1362
					28	706.564	457.622	1.5439	1.1066
					32	714.737	457.622	1.5618	1.0771
	0.6	36	0.075	3	20	685.121	457.622	1.4971	1.0778
					24	625.976	457.622	1.3678	1.0548
					28	656.061	457.622	1.4336	1.0318
					32	622.804	457.622	1.3609	1.0089
	0.7	42	0.075	3	20	608.248	457.622	1.3291	1.0009
					24	599.54	457.622	1.3101	0.9811
					28	560.41	457.622	1.2246	0.9614
					32	541.045	457.622	1.1822	0.9417
	0.8	48	0.075	3	20	552.879	457.622	1.2081	0.9299
					24	547.907	457.622	1.1972	0.9130
					28	546.596	457.622	1.1944	0.8962
					32	537.205	457.622	1.1739	0.8793

Table 3: Comparison of FE and Analytical Results for the $d/2R_1=0.075$ \$ - Stress Concentration Factor reported by H.E. Ang and C.L. Tan.

It is seen from the Table 2 & 3 that the SCF determined by FE approach and with analytical approach differ by 62 to 84%. This confirms that the analytical solution represented C.L.Tan and H.E.Ang may need some correction factors depend upon geometry of perforated disc. Hence, it can be suggested that the analytical approach may needs some refinement to predict the actual SCF in perforated disc.

5. DISCUSSION AND CONCLUSION

1. It is observed that as the number of holes increases the SCF decreases for most cases. But for some cases it is observed that the SCF increases abruptly after certain number of holes. This may be due to very less pitch width between the two consecutive holes. Thus there is a limit for the maximum number of holes to

keep the working stresses within safe limits.

2. As it is also seen that as the pitch circle radius of non-central holes increases, the SCF decreases. Thus the non-central holes very close to the central hole should be avoided in practice.
3. As the diameter of non-central holes increases the SCF decreases. This may be due to the smoothening of stress lines over larger curvature of hole. Thus larger diameter of non-central hole is preferred in actual practice.
4. Stress Concentration Factors derived for various geometric parameters like, R_2/R_1 , $d/2R_1$, $R_b/(R_2-R_1)$ and number of holes (N) can serve as a guideline for designing the perforated rotating disc.

5. REFERENCES

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